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The $SU(2) \otimes U(1)$ Electroweak Model based on the Nonlinearly Realized Gauge Group

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Abstract

The electroweak model is formulated on the nonlinearly realized gauge group $SU(2) \otimes U(1)$. This implies that in perturbation theory no Higgs field is present. The paper provides the effective action at the tree level, the Slavnov Taylor identity (necessary for the proof of physical unitarity), the local functional equation (used for the control of the amplitudes involving the Goldstone bosons) and the subtraction procedure (nonstandard, since the theory is not power-counting renormalizable). Particular attention is devoted to the number of independent parameters relevant for the vector mesons; in fact there is the possibility of introducing two mass parameters. With this choice the relation between the ratio of the intermediate vector meson masses and the Weinberg angle depends on an extra free parameter.

We briefly outline a method for dealing with γ_5 in dimensional regularization. The model is formulated in the Landau gauge for sake of simplicity and conciseness. The QED Ward identity has a simple and intriguing form.

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1 Introduction

Within the framework of the nonlinearly realized gauge theories of massive vector mesons, the extension of Yang-Mills theory from $SU(2)$ [1] to the $SU(2) \otimes U(1)$ group of the electroweak model [2] is far from being straightforward: the direction of the Spontaneous Symmetry Breaking (SSB) and the dependence of the tree-level action from this direction are non-trivial questions. The pure $SU(2)$ Yang-Mills theory [3], when a mass term is introduced by the nonlinearly realization of the gauge group, requires the use of transformations of *local* $SU(2)_L$ left and of *global* $SU(2)_R$ right; the local functional equation associated to the $SU(2)_L$ invariance together with the Weak Power Counting (WPC) allows to overcome the problem of nonrenormalizability and of the anonmalous interaction terms, while the $SU(2)_R$ selects a single symmetric mass term [1]. The introduction of a $U(1)$ associated to the hypercharge destroys the *global* $SU(2)_R$ right symmetry. Thus two mass invariants can be introduced: one for the neutral and one for the charged vector meson. Consequently the ratio of the vector meson masses is not fixed anymore by the Weinberg angle.

In this paper we start from the unique tree-level vertex functional of the theory based on the nonlinear realization of the $SU(2) \otimes U(1)$ gauge symmetry in the presence of the fermionic matter content of the Standard Model (with massless neutrinos) and compatible with the WPC. This provides the Feynman rules in terms of the tree level parameters and the overall mass scale Λ for the radiative corrections. A whole set of external sources are used in order to introduce a closed set of local operators necessary for the Becchi-Rouet-Stora-Tyutin (BRST) transformations [4], the local $SU(2)_L$ transformations and the Landau gauge fixing. The functional equations derived from the invariance of the path integral measure are the tools for the construction of the theory: the Slavnov-Taylor identities (STI) [5] in order to prove physical unitarity [6], [7], the Local Functional Equation (LFE) [8], [1] for the symmetric subtraction procedure and control of the tree-level couplings by the WPC [9], [1] and the Landau gauge equation.

It is understood that we approach the quantization by a series expansion in $\hbar$. Thus no elementary field for the Higgs Boson [10] is present in the theory, since the representation is nonlinear. The scenario is then very interesting. For instance an Higgs boson could emerge as a non-perturbative mechanism, but then its physical parameters are not constrained by the

radiative corrections of the low energy electroweak processes. Otherwise, our energy scale for the radiative corrections Λ is a manifestation of some other high energy physics.

The intention of the present note is to provide the theoretical basis and the technical tools for explicit calculations in the electroweak model based on the nonlinearly realized gauge group. Our subtraction procedure is based on minimal subtraction on specifically normalized amplitudes. Thus the presence of γ_5 poses serious problems to the whole strategy. Our approach is pragmatic. A new γ_D is introduced, which anticommutes with every γ_μ, and at the same time no statement is made about the analytic properties of the trace involving γ_D. Since the theory is not anomalous such traces never meet poles in $D-4$ and therefore we can impose that their limit for $D=4$ is continuous.

In this paper the Landau gauge is used for sake of simplicity and conciseness. Other covariant gauges are possible as discussed in ref. [11] for pure SU(2) massive Yang-Mills.

We postpone to a future publication the details in the derivation of the LFE [8],[12]. The use of LFE to establish the hierarchy among the Green functions and the control of the divergences in the limit $D=4$ are described in Refs. [8] and [13]. The geometrical aspects and the solutions of the linearized LFE can be found in [14]. The classical action is proposed in this paper, by using the criterion of WPC introduced for the nonlinear sigma model [9] and used in massive SU(2) Yang-Mills model [1]. The use of the Slavnov-Taylor identity in the case of Landau gauge in order to guarantee physical unitarity is discussed in ref. [7].

Some very important issues are not discussed here. In particular the connection of the present approach with previous attempts to remove the Higgs contribution in the large mass limit as in [15]. A similar approach for the nonlinear sigma model turns out to be exceedingly complex [16]. The issue of unitarity at large energy [17] when the Higgs field is removed (as in ref. [18]) is not discussed here.

In the present paper we are going to propose a subtraction procedure of the divergences which is unique; i.e. there are no extra free parameters originated by the subtraction procedure of the divergences, beside the mass scale of the radiative corrections Λ. Since the counterterms associated to finite renormalization cannot be reinserted back in the tree-level vertex functional

without violating either the symmetries or the WPC, we cannot perform on-shell finite renormalizations, since this implies finite counterterms at every order in the perturbative expansion. Moreover it turns out that in this scheme the symmetric formalism is the most practical one at variance with the formalism based on physical fields. In particular the use of the fields in the symmetric basis and of the Landau gauge yields a very simple and intruiging form for the Ward identity generated by the electric charge.

2 Preliminaries

We start from the classical action without gauge fixing and external sources for the composite operators. This allows a simpler discussion about the number of parameters and the constraint of WPC. In the next sections we shall introduce the gauge fixing and the external sources. Some details will be written in a formalism that generalizes the conventional notation.

$$
\begin{aligned}
&\Gamma^{(0)} = \Lambda^{(D-4)} \int d^D x \Bigg(2\,Tr\left\{ -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right\} \\
&+ M^2\, Tr\left\{ (g A_\mu - \frac{g'}{2} \Omega \tau_3 B_\mu \Omega^\dagger - F_\mu)^2 \right\} \\
&+ M^2 \frac{\kappa}{2} \Big(Tr\{ (g A_\mu - \frac{g'}{2} \Omega \tau_3 B_\mu \Omega^\dagger - F_\mu) \tau_3 \} \Big)^2 \\
&+ \sum_L \Big[\bar{L} (i\, \partial\!\!\!/ + g\, A\!\!\!/ + \frac{g'}{2} Y_L\, B\!\!\!/) L + \sum_R \bar{R} (i\, \partial\!\!\!/ + \frac{g'}{2} (Y_L + \tau_3)\, B\!\!\!/) R \Big] \\
&+ \sum_j \Big[m_{l_j}\, \bar{R}^l_j \frac{1-\tau_3}{2} \Omega^\dagger L^l_j - m_{q^u_j}\, \bar{R}^q_j \frac{1+\tau_3}{2} \Omega^\dagger L^q_j \\
&+ m_{q^d_k} V^\dagger_{kj}\, \bar{R}^q_k \frac{1-\tau_3}{2} \Omega^\dagger L^q_j + h.c. \Big] \Bigg)
\end{aligned}
\tag{1}
$$

where L and R are doublets such that

$$
\gamma_D L = -L \quad \gamma_D R = R,
\tag{2}
$$

being γ_D a gamma matrix that anticommutes with every other γ^μ. Y_L is the hypercharge of the left-fields. We use also the 2×2 matrix notation for the fields A_μ, Ω, F_μ

$$
A_\mu = A_{a\mu} \frac{\tau_a}{2} \qquad \Omega = \frac{1}{v} (\phi_0 + i \tau_a \phi_a), \quad \Omega \in SU(2)
$$

$$
\begin{aligned}
G_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \\
F_\mu &= i\Omega\partial_\mu\Omega^\dagger = F_{a\mu}\frac{\tau_a}{2} \\
F_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu .
\end{aligned} \tag{3}
$$

2.1 Fermions

The quark fields $(q_j^u, j = 1,2,3) = (u,c,t)$ and $(q_j^d, j = 1,2,3) = (d,s,b)$ are taken to be the mass eigenstates in the tree level lagrangian. Similar notation is used for the leptons $(l_j^u, j = 1,2,3) = (\nu_e, \nu_\mu, \nu_\tau)$ and $(l_j^d, j = 1,2,3) = (e,\mu,\tau)$. L is an element of the set of the left fields of the three families

$$
L \in \left\{ \begin{pmatrix} l^u_{Lj} \\ l^d_{Lj} \end{pmatrix}, \begin{pmatrix} q^u_{Lj} \\ V_{jk} q^d_{Lk} \end{pmatrix}, \quad j,k = 1,2,3 \right\}, \tag{4}
$$

where V_{jk} is the CKM matrix; the right components can also be written formally as doublets

$$
R \in \left\{ \begin{pmatrix} l^u_{Rj} \\ l^d_{Rj} \end{pmatrix}, \begin{pmatrix} q^u_{Rj} \\ q^d_{Rj} \end{pmatrix}, \quad j,k = 1,2,3 \right\} \tag{5}
$$

(color indices are not exhibited). The single left doublets are denoted by L_j^l, $j = 1,2,3$ for the leptons, L_j^q, $j = 1,2,3$ for the quarks.

2.2 $SU(2)$ left - and $U(1)$ right-local transformations

By making use of the path integral we shall derive some identities which stem both from the invariance of the integration measure over the fields and from the transformation properties of the action. If the action is not invariant, then one has to add new source terms coupled to the new generated operators. Thus we study the invariance properties of the functional in eq. (1) under the $SU(2)$ local transformations, where Ω is transformed on the left (thus we use the notation $SU(2)_L$)

$$
\begin{array}{ll}
\Omega' = U\Omega & B'_\mu = B_\mu \\
A'_\mu = UA_\mu U^\dagger + \frac{i}{g} U\partial_\mu U^\dagger & L' = UL \ . \\
F'_\mu = UF_\mu U^\dagger + iU\partial_\mu U^\dagger & R' = R
\end{array} \tag{6}
$$

Under the $\exp(i\frac{\alpha}{2}Y) \in U(1)$ local transformations Ω is transformed on the right (thus we use the notation $U(1)_R$)

$$
\begin{array}{ll}
e^{-i\frac{\alpha}{2}Y}\Omega e^{i\frac{\alpha}{2}Y} = \Omega V^\dagger & e^{-i\frac{\alpha}{2}Y}F_\mu e^{i\frac{\alpha}{2}Y} = F_\mu + i\Omega V^\dagger\partial_\mu V\Omega^\dagger \\
e^{-i\frac{\alpha}{2}Y}A_\mu e^{i\frac{\alpha}{2}Y} = A_\mu & e^{-i\frac{\alpha}{2}Y}B_\mu e^{i\frac{\alpha}{2}Y} = B_\mu + \frac{1}{g'}\partial_\mu\alpha \\
e^{-i\frac{\alpha}{2}Y}Le^{i\frac{\alpha}{2}Y} = \exp(i\frac{\alpha}{2}Y_L)L & e^{-i\frac{\alpha}{2}Y}Re^{i\frac{\alpha}{2}Y} = \exp(i\frac{\alpha}{2}(Y_L+\tau_3))R,
\end{array}
\tag{7}
$$

where Y_L is the hypercharge of the L and

$$
V(\alpha) = \exp(i\frac{\alpha}{2}\tau_3). \tag{8}
$$

The nonlinearity of the representation comes from the constraint

$$
\Omega\Omega^\dagger = 1 \Longrightarrow \phi_0^2 + \vec{\phi}^2 = v^2. \tag{9}
$$

The electric charge is defined as usual

$$
Q = I_3 + \frac{1}{2}Y \tag{10}
$$

where I_3 is a generator of $SU(2)_L$. Since the symmetry generated by Q is not spontaneously broken, then the component Ω_0 which acquires a non zero vacuum expectation value must obey the condition

$$
\langle 0|[Q,\Omega]|0\rangle = 0 \Longrightarrow \frac{1}{2}\tau_3\Omega_0 - \frac{1}{2}\Omega_0\tau_3 = 0. \tag{11}
$$

Out of the manifold that solves (11) we choose, at the tree level, the direction

$$
\phi_0 = \frac{1}{v}\sqrt{v^2 - \vec{\phi}^2}. \tag{12}
$$

The choice might not be stable under radiative corrections. However in our present approach (no Higgs and Landau gauge) the absence of tadpole graphs indicates that the vacuum expectation value of ϕ_0 receives no radiative corrections.

2.3 Two mass invariants and the WPC

The two expressions in eq. (1) multiplied by M^2 are invariant under $SU_L(2)\otimes U(1)_R$ transformations given by eqs. (6) and (7) [19]; in fact the bleached field

$$
w_\mu = \Omega^\dagger g A_\mu \Omega - g'\frac{\tau_3}{2}B_\mu + i\Omega^\dagger\partial_\mu\Omega \tag{13}
$$

is an $SU(2)_L$-invariant and transforms according to

$$w'_\mu = V w_\mu V^\dagger. \tag{14}$$

For each μ w_μ has four components. However, since

$$w_\mu^\dagger = w_\mu, \qquad Tr\{w_\mu\} = 0, \tag{15}$$

one gets

$$(w_\mu)_{11} = -(w_\mu)_{22} \qquad (w_\mu)^*_{12} = (w_\mu)_{21}. \tag{16}$$

Therefore the only two independent mass invariants are $(w_\mu)^2_{11}$ and $(w^\mu)_{21}(w_\mu)_{12}$.

Let us introduce the notation

$$\Phi \equiv \begin{pmatrix} i\phi_1 + \phi_2 \\ \phi_0 - i\phi_3 \end{pmatrix}, \quad \Phi^c \equiv i\tau_2\Phi^* \equiv \begin{pmatrix} \phi_0 + i\phi_3 \\ i\phi_1 - \phi_2 \end{pmatrix}; \tag{17}$$

hence

$$\Omega_{\alpha\beta} = \frac{1}{v}\Phi^c_\alpha \Phi_\beta. \tag{18}$$

From eqs. (6) and (7) one derives the transformation properties under $SU(2)_L \otimes U(1)_R$

$$\Phi' = U\Phi\ e^{-i\frac{\alpha}{2}}. \tag{19}$$

In eq. (1) the first mass invariant can be written in the form [20]

$$2\frac{M^2}{v^2}\left|\left(gA_\mu - \frac{g'}{2}B_\mu + i\partial_\mu\right)\Phi\right|^2 \tag{20}$$

while the second [19]

$$2\kappa\frac{M^2}{v^4}\left|\Phi^\dagger\left(gA_\mu - \frac{g'}{2}B_\mu + i\partial_\mu\right)\Phi\right|^2. \tag{21}$$

In the mass eigenstate basis given by

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(A_{1\mu} \mp iA_{2\mu}) \tag{22}$$

and

$$\begin{aligned} Z_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(gA_{3\mu} - g'B_\mu) \\ A_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(g'A_{3\mu} + gB_\mu), \end{aligned} \tag{23}$$

one finds

$$M_W^2 = g^2 M^2 \,, \qquad M_Z^2 = (1+\kappa)\frac{g^2 M^2}{c^2} \,, \tag{24}$$

while the photon A_μ is massless.

In the eq.(24) c is the cosine of the Weinberg angle θ_W. The latter is defined as usual according to

$$\tan\theta_W = \frac{g'}{g} \,. \tag{25}$$

The ratio of M_W and M_Z in eq.(24) is a function of the parameter κ. This is a peculiar feature of the nonlinearly realized electroweak model.

The invariance properties of the expressions in eqs. (20) and (21) are valid independently from the constraint (9). Thus the technique of bleaching for the construction of the $SU(2) \otimes U(1)$ invariants can be freely used also for the case of linear representation (Higgs mechanism). However, in renormalizable theories the term in eq. (21) is excluded being of dimension 6. The fermion sector can be considered in the same way by using the bleaching

$$\tilde{l}^d_{Lj} \equiv \frac{1}{v}\Phi^\dagger l_{Lj}, \quad \tilde{q}^u_{Lj} \equiv \frac{1}{v}\Phi^{c\dagger} q_{Lj}, \quad \tilde{q}^d_{Lj} \equiv \frac{1}{v}\Phi^\dagger q_{Lj}. \tag{26}$$

In our case (nonlinear representation of the gauge group) the generic graph with no external Goldstone boson legs (ancestor amplitudes) has a degree of superficial divergence bounded by

$$d(G) \le (D-2)n_L + 2 - N_B - N_F \tag{27}$$

where n_L is the number of loops and N_B, N_F the number of external gauge- and fermion-fields. Thus in the limit $D = 4$ the number of divergent one-particle-irreducible (1-PI) amplitudes is finite at fixed number of loops; while, if we consider also external Goldstone boson legs (descendant amplitudes), already at one loop the number of divergent (1-PI) amplitudes is infinite. It is interesting that the Fermions enter in eq. (27) with dimension 1 instead of the canonical $\frac{3}{2}$.

By imposing this constraint (WPC) one obtains that the number of independent and invariant terms in the action are those exhibited in eq. (1). Moreover it can be shown that our subtraction procedure, described in the sequel, does not destroy the bound given in eq. (27). Thus the WPC becomes a very important and efficient tool in the construction of the classical

action of the model. In particular the action is protected against anomalous couplings, which are present if one relays only on symmetry requirements [21].

3 Quantization, gauge fixing, ST, LFE, Landau gauge equation

The classical action in eq. (1) is the starting point of a complex strategy that takes into account the field quantization (which we perform with the tool of a gauge fixing). Due to the presence of unphysical modes, one has to introduce some Faddeev-Popov ghosts by requiring BRST invariance. The STI then translate on the Feynman amplitudes the BRST invariance and provide the necessary relations that guarantees the cancellation of the contributions of the unphysical modes for physical amplitudes. A further local functional equation (LFE) is necessary to account for the fact that the Goldstone modes enter as gauge modes in the model. The LFE allows to trace the correct subtraction procedure for the divergences and moreover it guarantees full hierarchy, i.e. all amplitudes involving the Goldstone boson (descendant amplitudes) are derived from the ancestor amplitudes (i.e. with no Goldstone bosons). The Landau gauge equation follows from the gauge fixing term and it is equivalent to the anti-ghost equation by making use of the STI.

According to this procedure we start from the classical action, add the gauge fixing terms and the Faddeev-Popov ghosts in order to implement the BRST transformations. We have to introduce all the source terms necessary for the renormalization of the new necessary composite operators. When we consider the $SU(2)_L \otimes U(1)_R$ transformations, again new composite operators emerge. We provide a set of source terms that is closed under the combined set of transformations. The complete analysis is left to a future work. Here we give the final results.

The tree level effective action describing the gauge fixing (Landau gauge) and the composite operator source terms is (in the notation we indicate with a * the sources necessary for the formulation of the STI: the anti-fields.)

$$\Gamma^{(0)}_{\rm GF}$$
$$= \Lambda^{(D-4)} \int d^D x \Bigg(b_0 \partial_\mu B^\mu - \bar{c}_0 \Box c_0 + 2Tr \Big\{ b \partial_\mu A^\mu - \bar{c} \partial^\mu D[A]_\mu c$$

$$+V^\mu \left(D[A]_\mu b - ig\bar{c}D[A]_\mu c - ig(D[A]_\mu c)\bar{c}\right) + \Theta^\mu \; D[A]_\mu \bar{c} \Big\} + K_0\phi_0$$

$$+A^*_{a\mu}\mathfrak{s}A^\mu_a + \phi^*_0\mathfrak{s}\phi_0 + \phi^*_a\mathfrak{s}\phi_a + c^*_a\mathfrak{s}c_a + \sum_L \left(L^*\mathfrak{s}L + \bar{L}^*\mathfrak{s}\bar{L}\right)\Bigg), \qquad (28)$$

where the Lagrange multipliers and the ghosts of $SU(2)_L$ are in matrix notation

$$\begin{aligned} & b = b_a\frac{1}{2}\tau_a, \qquad c = c_a\frac{1}{2}\tau_a, \qquad \bar{c} = \bar{c}_a\frac{1}{2}\tau_a \\ & D[A]_\mu \; c = \partial_\mu \; c - ig[A_\mu, c] \\ & = \frac{1}{2}\tau_a(\partial_\mu\delta_{ab} - g\epsilon_{abc}A_{c\mu})c_b. \end{aligned} \qquad (29)$$

The full effective action at the tree level (eqs. (1) and (28)) is invariant under the BRST transformations (not counting the source terms)

$$\begin{array}{llll} \mathfrak{s}A_\mu = D[A]_\mu \; c & \mathfrak{s}\Omega = ig \; c \; \Omega & \mathfrak{s}\bar{c} = b & \mathfrak{s}\bar{c}_0 = 0 \\ \mathfrak{s}c = ig \; c \; c & \mathfrak{s}B_\mu = 0 & \mathfrak{s}b = 0 & \mathfrak{s}b_0 = 0 \\ \mathfrak{s}L = igcL & \mathfrak{s}R = 0 & \mathfrak{s}c_0 = 0 & \end{array} \qquad (30)$$

$$\begin{array}{llll} \mathfrak{s}_1 A_\mu = 0 & \mathfrak{s}_1\Omega = -\frac{i}{2}g'\Omega c_0\tau_3 & \mathfrak{s}_1\bar{c} = 0 & \mathfrak{s}_1\bar{c}_0 = b_0 \\ \mathfrak{s}_1 c = 0 & \mathfrak{s}_1 B_\mu = \partial_\mu c_0 & \mathfrak{s}_1 b = 0 & \mathfrak{s}_1 b_0 = 0 \\ \mathfrak{s}_1 L = \frac{i}{2}g'c_0Y_L L & \mathfrak{s}_1 R = \frac{i}{2}g'c_0(Y_L+\tau_3)R & \mathfrak{s}_1 c_0 = 0. & \end{array} \qquad (31)$$

By construction

$$\{\mathfrak{s}, \mathfrak{s}_1\} = 0. \qquad (32)$$

No sources for the $\mathfrak{s}_1$ transforms of fields are used, since c_0 is a free field.

From the BRST transformation (30) we get the STI [4]

$$\mathcal{S}\Gamma \equiv \int d^Dx \Bigg[\Lambda^{-(D-4)}\Big(\Gamma_{A^*_{a\mu}}\Gamma_{A^\mu_a} + \Gamma_{\phi^*_a}\Gamma_{\phi_a} + \Gamma_{c^*_a}\Gamma_{c_a}$$

$$+\Gamma_{L^*}\Gamma_L + \Gamma_{\bar{L}^*}\Gamma_{\bar{L}}\Big) + b_a\Gamma_{\bar{c}_a} + \Theta_{a\mu}\Gamma_{V_{a\mu}} - K_0\Gamma_{\phi^*_0}\Bigg] = 0. \qquad (33)$$

The classical linearized form of the operator is

$$\mathcal{S}_0\Gamma \equiv \int d^Dx \Bigg[\Lambda^{-(D-4)}\Big(\Gamma^{(0)}_{A^\mu_a}\frac{\delta}{\delta A^*_{a\mu}} + \Gamma^{(0)}_{A^*_{a\mu}}\frac{\delta}{\delta A^\mu_a} + \Gamma^{(0)}_{\phi^*_a}\frac{\delta}{\delta\phi_a} + \Gamma^{(0)}_{\phi_a}\frac{\delta}{\delta\phi^*_a}$$

[4] The notation is as follows: Γ_ψ stands for $\delta\Gamma/\delta\psi$, while W_ψ for $\delta W/\delta J_\psi$, with ψ any of the quantized fields of the model and J_ψ its source. The connected generating functional $W[J]$ is related to the vertex functional $\Gamma[\psi]$ by $W = \Gamma + \int d^Dx\, J\psi$.

$$
\begin{aligned}
&+\Gamma^{(0)}_{c^*_a}\frac{\delta}{\delta c_a}+\Gamma^{(0)}_{c_a}\frac{\delta}{\delta c^*_a}+\Gamma^{(0)}_{L^*}\frac{\delta}{\delta L}+\Gamma^{(0)}_{L}\frac{\delta}{\delta L^*}\\
&+\Gamma^{(0)}_{\bar L^*}\frac{\delta}{\delta \bar L}+\Gamma^{(0)}_{\bar L}\frac{\delta}{\delta \bar L^*}\Big)+b_a\frac{\delta}{\delta \bar c_a}+\Theta_{a\mu}\frac{\delta}{\delta V_{a\mu}}-K_0\frac{\delta}{\delta \phi^*_0}\Bigg]\Gamma. \qquad (34)
\end{aligned}
$$

In both eqs. (33) and (34) the sum over L and $\bar L^*$ is understood over the components explicitly shown in eq. (4). From the transformations in eq. (31) we get the relation

$$
\begin{aligned}
&\frac{2}{g'}\Box b_0-\frac{2}{g'}\partial^\mu\frac{\delta\Gamma}{\delta B^\mu}+\Lambda^{(D-4)}\phi_3K_0+\phi_2\frac{\delta\Gamma}{\delta\phi_1}-\phi_1\frac{\delta\Gamma}{\delta\phi_2}-\frac{1}{\Lambda^{(D-4)}}\frac{\delta\Gamma}{\delta K_0}\frac{\delta\Gamma}{\delta\phi_3}\\
&-\phi^*_3\frac{\delta\Gamma}{\delta\phi^*_0}+\phi^*_2\frac{\delta\Gamma}{\delta\phi^*_1}-\phi^*_1\frac{\delta\Gamma}{\delta\phi^*_2}+\phi^*_0\frac{\delta\Gamma}{\delta\phi^*_3}\\
&+iY_LL\frac{\delta\Gamma}{\delta L}-iY_L\bar L\frac{\delta\Gamma}{\delta\bar L}+i(Y_L+\tau_3)R\frac{\delta\Gamma}{\delta R}-i\bar R(Y_L+\tau_3)\frac{\delta\Gamma}{\delta\bar R}\\
&-iY_LL^*\frac{\delta\Gamma}{\delta L^*}+iY_L\bar L^*\frac{\delta\Gamma}{\delta\bar L^*}=0.
\end{aligned} \qquad (35)
$$

The same relation is obtained by using the $U(1)_R$ transformations of eq. (7), complemented with the following extension to the new variables

$$
\begin{array}{llll}
V'_\mu=V_\mu & \Omega^{*'}=V\Omega^* & L^{*'}=\exp(-i\frac{\alpha}{2}Y_L)L^* & K'_0=K_0\\
\Theta'_\mu=\Theta_\mu & b'=b & \bar L^{*'}=\exp(i\frac{\alpha}{2}Y_L)\bar L^* & b'_0=b_0\\
c'=c & \bar c'=\bar c & c^{*'}=c^* & \\
c'_0=c_0 & \bar c'_0=\bar c_0 & A^{*'}_\mu=A^*_\mu. &
\end{array} \qquad (36)
$$

The Landau gauge equation is

$$
\Gamma_{b_a}=\Lambda^{(D-4)}\Big(D^\mu[V](A_\mu-V_\mu)\Big)_a \qquad (37)
$$

which implies the ghost equation

$$
\Gamma_{\bar c_a}=\Big(-D_\mu[V]\Gamma_{A^*_\mu}+\Lambda^{(D-4)}D_\mu[A]\Theta^\mu\Big)_a, \qquad (38)
$$

by using the STI (33).

Now we explore the LFE that follows from the invariance of the path integral measure over the transformations (6). In doing that we have first to extend the transformations to the newly introduced fields and sources. This is straightforward by following the criterion that all the transformations should close on a finite number of composite operators. Thus eq. (6) is

complemented by

$$
\begin{array}{llll}
V'_\mu = UV_\mu U^\dagger + \frac{i}{g}U\partial_\mu U^\dagger & \Omega^{*'} = \Omega^* U^\dagger & L^{*'} = L^* U^\dagger & K'_0 = K_0 \\
\Theta'_\mu = U\Theta_\mu U^\dagger & b' = UbU^\dagger & \bar{L}^{*'} = U\bar{L}^* & b'_0 = b_0 \\
c' = UcU^\dagger & \bar{c}' = U\bar{c}U^\dagger & c^{*'} = c^* & \\
c'_0 = c_0 & \bar{c}'_0 = \bar{c}_0 & A^{*'}_\mu = UA^*_\mu U^\dagger . &
\end{array} \tag{39}
$$

Thus the resulting identity associated to the $SU(2)_L$ local transformations is (x-dependence is not shown)

$$
\begin{aligned}
&(\mathcal{W}\Gamma)_a \equiv -\frac{1}{g}\partial_\mu \Gamma_{V_{a\mu}} + \epsilon_{abc}V_{c\mu}\Gamma_{V_{b\mu}} - \frac{1}{g}\partial_\mu\Gamma_{A_{a\mu}} \\
&+\epsilon_{abc}A_{c\mu}\Gamma_{A_{b\mu}} + \epsilon_{abc}b_c\Gamma_{b_b} + \frac{\Lambda^{(D-4)}}{2}K_0\phi_a + \frac{1}{2\Lambda^{(D-4)}}\Gamma_{K_0}\Gamma_{\phi_a} \\
&+\frac{1}{2}\epsilon_{abc}\phi_c\Gamma_{\phi_b} + \epsilon_{abc}\bar{c}_c\Gamma_{\bar{c}_b} + \epsilon_{abc}c_c\Gamma_{c_b} \\
&+\frac{i}{2}\tau_a L\Gamma_L - \frac{i}{2}\bar{L}\tau_a\Gamma_{\bar{L}} - \frac{i}{2}L^*\tau_a\Gamma_{L^*} + \frac{i}{2}\tau_a\bar{L}^*\Gamma_{\bar{L}^*} \\
&+\epsilon_{abc}\Theta_{c\mu}\Gamma_{\Theta_{b\mu}} + \epsilon_{abc}A^*_{c\mu}\Gamma_{A^*_{b\mu}} + \epsilon_{abc}c^*_c\Gamma_{c^*_b} - \frac{1}{2}\phi^*_0\Gamma_{\phi^*_a} \\
&+\frac{1}{2}\epsilon_{abc}\phi^*_c\Gamma_{\phi^*_b} + \frac{1}{2}\phi^*_a\Gamma_{\phi^*_0} = 0\,,
\end{aligned} \tag{40}
$$

where the nonlinearity of the realization of the $SU(2)_L$ gauge group is revealed by the presence of the bilinear term $\Gamma_{K_0}\Gamma_{\phi_a}$. This is essential for the hierarchy; in fact eq. (40) shows that every amplitude with $\phi-$external leg (descendant amplitudes) can be obtained from those without. When this property is supplemented with the WPC, it appears that the number of independent counterterms necessary to make the theory is finite at every order of the perturbative expansion in loops. A quick inspection to eq. (40) shows that one has to evaluate a whole set of amplitudes with all possible external sources in order to fix the descendant amplitudes. Moreover equation (27) have to be updated to the presence of the external sources. A straightforward argument shows that the superficial degree of divergence of an ancestor amplitude is bounded by

$$
\begin{aligned}
d(\mathcal{G}) \leq (D-2)n + 2 - N_A - N_B - N_c - N_F - N_{\bar{F}} - N_V - N_{\phi^*_a} \\
- 2(N_\Theta + N_{A^*} + N_{\phi^*_0} + N_{L^*} + N_{\bar{L}^*} + N_{c^*} + N_{K_0})\,.
\end{aligned} \tag{41}
$$

where N_X is the number of external fields $X = A_\mu, B_\mu, b, b_0, L, R$ and sources $X = V_\mu, \Theta_\mu, A^*, L^*, \bar{L}^*, c^*, K_0$. In passing it is worth noticing that the STI are not sufficient to fix all the descendant amplitudes. This feature is present

also in pure $SU(2)$ massive Yang-Mills [1]. In conclusion, the LFE (40) is the right tool to describe at the quantum level the *gauge* character of the Goldstone boson fields $\vec{\phi}$ and how, through the hierarchy and the bleaching technique, they can be managed.

The classical linearized form of the operator in eq. (40) is

$$
\begin{aligned}
(\mathcal{W}_0\Gamma)_a \equiv \Bigg(& -\frac{1}{g}\partial_\mu \frac{\delta}{\delta V_{a\mu}} + \epsilon_{abc} V_{c\mu}\frac{\delta}{\delta V_{b\mu}} - \frac{1}{g}\partial_\mu \frac{\delta}{\delta A_{a\mu}} \\
& + \epsilon_{abc} A_{c\mu}\frac{\delta}{\delta A_{b\mu}} + \epsilon_{abc} b_c \frac{\delta}{\delta b_b} + \frac{1}{2\Lambda^{(D-4)}}\frac{\delta\Gamma^{(0)}}{\delta K_0}\frac{\delta}{\delta\phi_a} \\
& + \frac{1}{2\Lambda^{(D-4)}}\frac{\delta\Gamma^{(0)}}{\delta\phi_a}\frac{\delta}{\delta K_0} + \frac{1}{2}\epsilon_{abc}\phi_c\frac{\delta}{\delta\phi_b} + \epsilon_{abc}\bar{c}_c\frac{\delta}{\delta\bar{c}_b} + \epsilon_{abc}c_c\frac{\delta}{\delta c_b} \\
& + \frac{i}{2}\tau_a L\frac{\delta\Gamma}{\delta L} - \frac{i}{2}\bar{L}\tau_a\frac{\delta\Gamma}{\delta\bar{L}} - \frac{i}{2}L^*\tau_a\frac{\delta\Gamma}{\delta L^*} + \frac{i}{2}\tau_a\bar{L}^*\frac{\delta\Gamma}{\delta\bar{L}^*} \\
& + \epsilon_{abc}\Theta_{c\mu}\frac{\delta}{\delta\Theta_{b\mu}} + \epsilon_{abc}A^*_{c\mu}\frac{\delta}{\delta A^*_{b\mu}} + \epsilon_{abc}c^*_c\frac{\delta}{\delta c^*_b} - \frac{1}{2}\phi^*_0\frac{\delta}{\delta\phi^*_a} \\
& + \frac{1}{2}\epsilon_{abc}\phi^*_c\frac{\delta}{\delta\phi^*_b} + \frac{1}{2}\phi^*_a\frac{\delta}{\delta\phi^*_0}\Bigg)\Gamma\,.
\end{aligned} \tag{42}
$$

It is straightforward to prove that

$$
[\mathcal{S}_0, \mathcal{W}_0] = 0. \tag{43}
$$

4 Subtraction Strategy

The superficial degree of divergence in eqs. (27) or (41) shows that the theory is not renormalizable even if we invoke the hierarchy in order to renormalize only the ancestor amplitudes. This item has been considered at length by the present authors. The extensive discussion is in Ref. [12], where we argue in favor of a particular subtraction procedure which respects locality and unitarity at variance with the algebraic renormalization which cannot be implemented in the present case.

To ferret out the procedure of the removal of divergences, eq. (40) is used. Dimensional regularization provides the most natural environment. Let us denote by

$$
\Gamma^{(n,k)} \tag{44}
$$

the vertex functional for 1-particle irreducible amplitudes at n- order in loops where the countertems enter with a total power k in $\hbar$. In dimensional

regularization we can perform a grading in k of eq. (40). Thus if we have successfully performed the subtraction procedure satisfying eq. (40) up to order $n-1$ the next order effective action

$$\Gamma^{(n)} = \sum_{k=0}^{n-1} \Gamma^{(n,k)} \tag{45}$$

violates eq. (40) since the counterterm $\hat{\Gamma}^{(n)}$ is missing. The breaking term can be determined by writing eq. (40) at order n at the grade $k \leq n-1$ and then by summing over k. One gets

$$\begin{aligned} &\mathcal{W}_0 \Gamma^{(n)} + \frac{1}{2\Lambda^{(D-4)}} \sum_{n'=1}^{n-1} \Big(\frac{\delta\Gamma^{(n-n')}}{\delta K_0}\Big)\Big(\frac{\delta\Gamma^{(n')}}{\delta\phi_a}\Big) \\ &= \frac{1}{2\Lambda^{(D-4)}} \sum_{n'=1}^{n-1} \Big(\frac{\delta\Gamma^{(n-n',n-n')}}{\delta K_0}\Big)\Big(\frac{\delta\Gamma^{(n',n')}}{\delta\phi_a}\Big). \end{aligned} \tag{46}$$

The first term in the LHS of eq. (46) has pole parts in $D-4$ while the second is finite, since the factors are of order $< n$, thus already subtracted. The breaking term contains only counterterms $\hat{\Gamma}^j = \Gamma^{(j,j)}$, $j < n$. This suggests the Ansatz that the finite part of the Laurent expansion at $D = 4$

$$\frac{1}{\Lambda^{(D-4)}} \Gamma^{(n)} \tag{47}$$

gives the correct prescription for the subtraction of the divergences; i.e. one has to divide both members of eq. (46) by $\Lambda^{(D-4)}$ and remove only the pole parts (minimal subtraction). Thus the counterterms have the form

$$\hat{\Gamma}^{(n)} = \Lambda^{(D-4)} \int \frac{d^D x}{(2\pi)^D} \mathcal{M}^{(n)}(x) \tag{48}$$

where the integrand is a local formal power series in the fields, the external sources and their derivatives (a local polynomial as far as the ancestor monomials are concerned) and it possesses only pole parts in its Laurent expansion at $D = 4$.

In conclusion the subtraction procedure relies on dimensional regularization and it allows only one extra free parameter: Λ.

In practice there are two ways to proceed in the regularization procedure. One can use the forest formula and use minimal subtraction for every (properly normalized) subgraph. It is possible, as alternative, to evaluate the counterterms for the ancestor amplitudes and then obtain from those all the necessary counterterms involving the Goldstone boson fields $\vec{\phi}$.

4.1 γ_5

Our subtraction procedure is based on minimal subtraction on specifically normalized amplitudes. Thus the presence of γ_5 poses serious problems to the whole strategy. Our approach is pragmatic. We introduce a new γ_D which anticommutes with every γ_μ and at the same time no statement is made about the analytic properties of the trace involving γ_D. Since the theory is not anomalous such traces never meet poles in $D-4$ and therefore we can evaluate the traces at $D=4$.

5 Physical observables

The classical action of the nonlinearly realized electroweak model (for gauge- and fermion fields) has been modified in order to introduce mass term invariants. Moreover new fields $(c, \bar{c}, c_0, \bar{c}_0, b, b_0)$ and sources have been added in order to perform the quantization and to establish the tools necessary for the removal of divergences. The physical interpretation of the model has to go through the standard selection of the physical modes based on the Slavnov-Taylor linearized operator $\mathcal{S}_0$ in eq. (34). The unphysical modes are the Faddeev-Popov ghosts, the scalar components of the massive vector mesons and of the photon and the Goldstone bosons. All these modes have to conspire in order to give zero contribution to the unitarity equation for physical states. The STI is the essential tool in order to guarantee that the theory has the right unitarity property [7]. Beside the fields, few sources and parameters have been introduced. It is established in eq. (34) that these auxiliary sources come in doublets [22],[23]

$$\begin{aligned}
&\mathcal{S}_0 V_\mu = \Theta_\mu \\
&\mathcal{S}_0 \phi_0^* = -K_0 \\
&\mathcal{S}_0 \bar{c} = b.
\end{aligned} \tag{49}$$

It can be shown that any physical relevant amplitude can be separated into a part, where the doublets are absent, plus terms that are irrelevant for the physical S-matrix elements. These results are part of common knowledge for the Lagrange multiplier b [24] and for the background gauge field V_μ [25]. It is a bit surprising that the source K_0, coupled to the order parameter field ϕ_0, is an unphysical object.

The above argument shows also that the constant v introduced in eq. (3) is not a physical parameter. In fact it is removed from the tree level effective action by rescaling $\vec{\phi} \to v\vec{\phi}$ and $K_0 \to v^{-1}K_0$. The rescaling of $\vec{\phi}$ is of no effect since it is a path integral integration variable. The rescaling of K_0 has no effect on physical amplitudes since it is a Slavnov-Taylor doublet.

Λ is a genuine new parameter that provides the mass scale of the radiative corrections.

6 Electric charge and gauge invariance

Electric charge generates an exact symmetry, i.e. not spontaneously broken. The associated local identity for the generating functionals guarantees that longitudinal polarizations of the electromagnetic potential decouple from physical fields. It is worth to examine in detail the corresponding identity for the vertex functional.

$$
\begin{aligned}
&\frac{1}{g'}\Box b_0 + \Bigg(-\frac{1}{g'}\partial^\mu \frac{\delta}{\delta B^\mu} - \frac{1}{g}\partial_\mu \frac{\delta}{\delta A_{3\mu}} - \frac{1}{g}\partial_\mu \frac{\delta}{\delta V_{3\mu}} \\
&+A_{2\mu}\frac{\delta}{\delta A_{1\mu}} - A_{1\mu}\frac{\delta}{\delta A_{2\mu}} + iQL\frac{\delta}{\delta L} - i\bar{L}Q\frac{\delta}{\delta \bar{L}} + iQR\frac{\delta}{\delta R} - i\bar{R}Q\frac{\delta}{\delta \bar{R}} \\
&+\phi_2\frac{\delta}{\delta\phi_1} - \phi_1\frac{\delta}{\delta\phi_2} + b_2\frac{\delta}{\delta b_1} - b_1\frac{\delta}{\delta b_2} + c_2\frac{\delta}{\delta c_1} - c_1\frac{\delta}{\delta c_2} \\
&+\bar{c}_2\frac{\delta}{\delta\bar{c}_1} - \bar{c}_1\frac{\delta}{\delta\bar{c}_2} + V_{2\mu}\frac{\delta}{\delta V_{1\mu}} - V_{1\mu}\frac{\delta}{\delta V_{2\mu}} + \Theta_{2\mu}\frac{\delta}{\delta\Theta_{1\mu}} - \Theta_{1\mu}\frac{\delta}{\delta\Theta_{2\mu}} \\
&+A^*_{2\mu}\frac{\delta}{\delta A^*_{1\mu}} - A^*_{1\mu}\frac{\delta}{\delta A^*_{2\mu}} + \phi^*_2\frac{\delta}{\delta\phi^*_1} - \phi^*_1\frac{\delta}{\delta\phi^*_2} + c^*_2\frac{\delta}{\delta c^*_1} - c^*_1\frac{\delta}{\delta c^*_2} \\
&-iQL^*\frac{\delta}{\delta L^*} + i\bar{L}^*Q\frac{\delta}{\delta\bar{L}^*}\Bigg)\Gamma = 0\,,
\end{aligned}
\tag{50}
$$

where Q is the electric charge of the component of the multiplet. Equation (50) is very important since it establishes gauge invariance (here, the decoupling of the longitudinal polarizations of the photons from the physical S-matrix elements). It is remarkable that the identity is a linear operator in a theory where physical unitarity is guaranteed by BRST invariance, i.e. under nonlinear transformations; in fact the bilinear term $\Gamma_{K_0}\Gamma_{\phi_3}$ of eqs. (35) and (40) has cancelled out. Moreover it is surprising that the equation takes such a simple form in the symmetric notation $(A_{3\mu}, B_\mu)$. In fact, in

terms of the fields Z_μ, A_μ in eq.(23) the neutral boson part in eq. (50) takes the form

$$-\frac{1}{g'}\partial^\mu\frac{\delta}{\delta B^\mu}-\frac{1}{g}\partial_\mu\frac{\delta}{\delta A_{3\mu}}=-\frac{\sqrt{g^2+g'^2}}{gg'}\partial^\mu\frac{\delta}{\delta A^\mu}. \tag{51}$$

The term $-\frac{1}{g}\partial_\mu\frac{\delta}{\delta V_{3\mu}}$ takes into account that the field of the photon, as superposition of $(A_{3\mu}, B_\mu, \partial_\mu b_3)$, is modified by the perturbative corrections. The latter are not present if only insertions of $\mathcal{S}_0$-ivariant operators are considered, since $V_{a\mu}$, $\Theta_{a\mu}$ is a BRST doublet, as discussed in Section 5.

7 Conclusions

In the framework provided by the nonlinear realization of the gauge group $SU(2)\otimes U(1)$ for the electroweak model new features show up, very interesting both from the phenomenological and theoretical point of view. The model is nonrenormalizable; therefore the couplings with negative dimension are not excluded and moreover the standard tools for the subtraction of the divergences cannot be applied.

The discovery of a new LFE, which follows from the invariance of the path integral measure, shows the way for a unique subtraction strategy of the divergences without changing the number of tree-level parameters apart from a common mass scale of the radiative corrections. The algorithm is strictly connected with dimensional regularization and symmetric subtraction of the pole parts in the Laurent expansion of the 1-particle irreducible amplitudes.

The same equation provides a hierarchy for the amplitudes: those involving the Goldstone bosons can be derived from those without. Thus also the superficial degree of divergence of the graphs can be studied by means a new tool, the weak power counting. The number of independent divergent amplitudes turns out to be finite at any given loop order. We argued that this property is stable under the procedure of subtraction.

The weak power counting provides a way to solve the other problem of limiting the anomalous couplings. In our opinion this seems to be the only way to stabilize a nonrenormalizable model under the process of making the amplitudes finite. In the present case, $SU(2)\otimes U(1)$, two invariants appear that contribute to the mass of the vector mesons. Thus the simple tree-level relation among masses and Weinberg angle is not working.

The paper illustrates is a brief way all these points. We mention the problem of γ_5 in dimensional regularization, where an escape is permitted by the absence of any anomaly for the electroweak currents.

We use the Landau gauge and the fields in the symmetric basis. This choice brings unexpected simplifications in the notations and in the equations. Very interesting is the Ward identity associated to the existence of the electric charge.

References

[1] D. Bettinelli, R. Ferrari and A. Quadri, Phys. Rev. D **77**, 045021 (2008) [arXiv:0705.2339 [hep-th]].

[2] S. L. Glashow, Nucl. Phys. **22** (1961) 579.
S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).
A. Salam, "Weak And Electromagnetic Interactions," in *Elementary Particle Theory; Nobel Symposium No.8* ed. N. Svartholm (Almqvist and Wiksell, Stockholm 1968).

[3] C. N. Yang and R. L. Mills, Phys. Rev. **96** (1954) 191.

[4] C. Becchi, A. Rouet and R. Stora, Commun. Math. Phys. **42** (1975) 127; Annals Phys. **98** (1976) 287; I. V. Tyutin, Lebedev Institute preprint N39 (1975).

[5] A. A. Slavnov, Theor. Math. Phys. **10** (1972) 99 [Teor. Mat. Fiz. **10** (1972) 153]; J. C. Taylor, Nucl. Phys. B **33** (1971) 436.

[6] G. Curci and R. Ferrari, Nuovo Cim. A **35** (1976) 273; T. Kugo and I. Ojima, Phys. Lett. B **73** (1978) 459.

[7] R. Ferrari and A. Quadri, JHEP **0411** (2004) 019 [arXiv:hep-th/0408168].

[8] R. Ferrari, JHEP **0508** (2005) 048 [arXiv:hep-th/0504023].

[9] R. Ferrari and A. Quadri, Int. J. Theor. Phys. **45** (2006) 2497 [arXiv:hep-th/0506220].

[10] P. W. Higgs, Phys. Lett. **12** (1964) 132; Phys. Rev. Lett. **13** (1964) 508; Phys. Rev. **145** (1966) 1156; F. Englert and R. Brout, Phys. Rev. Lett. **13** (1964) 321; G. S. Guralnik, C. R. Hagen and T. W. B. Kibble, Phys. Rev. Lett. **13** (1964) 585; T. W. B. Kibble, Phys. Rev. **155** (1967) 1554.

[11] D. Bettinelli, R. Ferrari and A. Quadri, arXiv:0712.1410 [hep-th].

[12] D. Bettinelli, R. Ferrari and A. Quadri, Int. J. Mod. Phys. A **23**, 211 (2008) [arXiv:hep-th/0701197].

[13] R. Ferrari and A. Quadri, JHEP **0601** (2006) 003 [arXiv:hep-th/0511032].

[14] D. Bettinelli, R. Ferrari and A. Quadri, JHEP **0703** (2007) 065 [arXiv:hep-th/0701212].

[15] S. Dittmaier and C. Grosse-Knetter, Phys. Rev. D **52** (1995) 7276 [arXiv:hep-ph/9501285].

[16] D. Bettinelli, R. Ferrari and A. Quadri, Int. J. Theor. Phys. **46** (2007) 2560 [arXiv:hep-th/0611063].

[17] M. Froissart, Phys. Rev. **123** (1961) 1053.

[18] B. W. Lee, C. Quigg and H. B. Thacker, Phys. Rev. D **16**, 1519 (1977).

[19] T. Appelquist and C. W. Bernard, Phys. Rev. D **22**, 200 (1980); A. C. Longhitano, Phys. Rev. D **22**, 1166 (1980); Nucl. Phys. B **188**, 118 (1981); R. D. Peccei and X. Zhang, Nucl. Phys. B **337**, 269 (1990); R. Ferrari, M. Picariello and A. Quadri, Phys. Lett. B**611**, 215 (2005) [arXiv:hep-th/0409194].

[20] Mass terms *à la* Stückelberg (E. C. G. Stückelberg, Helv. Phys. Acta **11** (1938) 299; H. Ruegg and M. Ruiz-Altaba, Int. J. Mod. Phys. A **19** (2004) 3265 [arXiv:hep-th/0304245]) have been considered since long. See: D. G. Boulware, Annals Phys. **56** (1970) 140; A. A. Slavnov, Teor. Mat. Fiz. **10** (1972) 305; Yu. N. Kafiev, Nucl. Phys. B **201** (1982) 341.

[21] R. D. Peccei and X. Zhang in [19].

[22] G. Barnich, F. Brandt and M. Henneaux, Phys. Rept. **338** (2000) 439 [arXiv:hep-th/0002245].

[23] A. Quadri, JHEP **0205** (2002) 051 [arXiv:hep-th/0201122].

[24] O. Piguet and S. P. Sorella, "Algebraic renormalization: Perturbative renormalization, symmetries and anomalies," Lect. Notes Phys. **M28** (1995) 1.

[25] L. F. Abbott, Nucl. Phys. B **185** (1981) 189; L. F. Abbott, M. T. Grisaru and R. K. Schaefer, Nucl. Phys. B **229** (1983) 372; P. A. Grassi, Nucl. Phys. B **462** (1996) 524 [arXiv:hep-th/9505101]; P. A. Grassi, Nucl. Phys. B **537** (1999) 527 [arXiv:hep-th/9804013]; A. Denner, G. Weiglein and S. Dittmaier, Nucl. Phys. B **440** (1995) 95 [arXiv:hep-ph/9410338]; C. Becchi and R. Collina, Nucl. Phys. B **562** (1999) 412 [arXiv:hep-th/9907092]; R. Ferrari, M. Picariello and A. Quadri, Annals Phys. **294** (2001) 165 [arXiv:hep-th/0012090].